

AD-A250 389



NPS-PH-92-007

# NAVAL POSTGRADUATE SCHOOL

Monterey, California



DTIC  
ELECTE  
MAY 26 1992  
S B D

Amending the w. Velocity Scale for  
Surface Layer, Entrainment Zone, and  
Baroclinic Shear in Mixed Forced/Free  
Turbulent Convection

by

R.F. Kamada

March 1992

Final Report for Period  
October 1990 - September 1991

Approved for public release; distribution unlimited

Prepared for: U.S. Air Force, Space Division,  
Los Angeles AFB, California 90009

92-13744



NAVAL POSTGRADUATE SCHOOL  
Monterey, California


Rear Admiral R. W. West  
Superintendent

H. Shull  
Provost


The work reported herein was prepared for and funded by U.S. Air Force Space Systems Division, Los Angeles, CA, 90009.

Reproduction of all or part of this document is authorized.

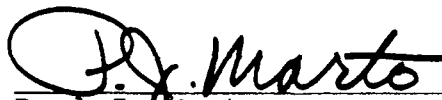
This report was prepared by

  
R. F. KAMADA,  
Adjunct Research  
Professor of Physics

Reviewed by:

  
K. E. Woehler,  
Chairman,  
Department of Physics

Released by:

  
Paul J. Marto,  
Dean of Research

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

## REPORT DOCUMENTATION PAGE

Form Approved  
GMB No 0704-0188

|   |   |  |                         |
|---|---|--|-------------------------|
| 1a REPORT SECURITY CLASSIFICATION<br>UNCLASSIFIED   |   | 1b RESTRICTIVE MARKINGS  |                         |
| 2a SECURITY CLASSIFICATION AUTHORITY  |   | 3 DISTRIBUTION AVAILABILITY OF REPORT<br>approved for public release; distribution unlimited.        |                         |
| 2b DECLASSIFICATION/DOWNGRADING SCHEDULE  |   |  |                         |
| 4 PERFORMING ORGANIZATION REPORT NUMBER(S)<br>NPS-PH-92-007   |   | 5 MONITORING ORGANIZATION REPORT NUMBER(S)   |                         |
| 6a NAME OF PERFORMING ORGANIZATION  | 6b OFFICE SYMBOL<br>(if applicable)             | 7a NAME OF MONITORING ORGANIZATION<br>USAF Space Systems Division                                    |                         |
| 6c ADDRESS (City, State, and ZIP Code)  |   | 7b ADDRESS (City, State, and ZIP Code)<br>Los Angeles AFB, CA 90009                                  |                         |
| 8a NAME OF FUNDING SPONSORING ORGANIZATION<br>USAF Space Systems Div.   | 8b OFFICE SYMBOL<br>(if applicable)<br>USAF/SSD | 9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER<br>MPIR FY76169100412                                 |                         |
| 8c ADDRESS (City, State, and ZIP Code)<br>Los Angeles AFB, CA 90009   |   | 10 SOURCE OF FUNDING NUMBERS   |                         |
|   |   | PROGRAM ELEMENT NO   | PROJECT NO              |
|   |   | TASK NO  | WORK UNIT ACCESSION NO  |
| 11 TITLE (Include Security Classification)<br>AMENDING THE $w_*$ VELOCITY SCALE FOR SURFACE LAYER, ENTRAINMENT ZONE, AND BAROCLINIC SHEAR IN MIXED FORCED/FREE TURBULENT CONVECTION.  |   |  |                         |
| 12 PERSONAL AUTHOR(S)<br>R. F. Kamada   |   |  |                         |
| 13a TYPE OF REPORT<br>technical   | 13b TIME COVERED<br>FROM 10/90 TO 9/91          | 14 DATE OF REPORT (Year, Month, Day)<br>920330   | 15 PAGE COUNT<br>14     |
| 16 SUPPLEMENTARY NOTATION   |   |  |                         |
| 17 COSATI CODES   |   | 18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)                     |                         |
| FIELD   | GROUP   | SUB-GROUP  |                         |
|   |   |  |                         |
|   |   |  |                         |
|   |   |  |                         |
|   |   | Convective Boundary layer, similarity, forced and free convection scaling, turbulent velocity scale. |                         |
| 19 ABSTRACT (Continue on reverse if necessary and identify by block number)<br>We amend the free convective mixed layer velocity scale, $w_*$ to $w_{*s}$ to include mechanical turbulence from forced convection. $w_{*s}$ has discrete surface layer, entrainment zone, and baroclinic components and requires the length scales: $z_0$ , $L$ , $\Delta h$ , $z_i$ , and $z_b$ , a newly defined baroclinic length. To close the system we offer simple models for $\Delta h$ , the entrainment zone depth. The $\Delta h$ models show realistic diurnal hysteresis, using a critical bulk Richardson number which exceeds unity. |   |  |                         |
| 20 DISTRIBUTION AVAILABILITY OF ABSTRACT<br><input checked="" type="checkbox"/> UNCLASSIFIED, UNLIMITED <input type="checkbox"/> SAME AS RPT <input type="checkbox"/> DTIC USERS  |   | 21 ABSTRACT SECURITY CLASSIFICATION<br>UNCLASSIFIED  |                         |
| 22a NAME OF RESPONSIBLE INDIVIDUAL<br>R. F. Kamada  |   | 22b TELEPHONE (Include Area Code)<br>(408) 646-2674  | 22c OFFICE SYMBOL<br>PH |

## INTRODUCTION

Deardorff's (1972) similarity theory for convective boundary layers (CBL)s suggests that CBL parameters are controlled by buoyancy,  $\beta \equiv g/\theta$ , (where  $g$  is gravitational acceleration and  $\theta$  is potential temperature), surface temperature flux,  $w'\theta'_0$ , height  $z$  above the surface, and  $z_i$ , the height of the inversion base which marks the top of the mixing-layer (ML). The velocity, temperature, and humidity scales:  $w_*$ ,  $u_*^{ML}$ ,  $\theta_*^{ML}$ ,  $q_*^{ML}$ , formed from these parameters are used in universal relationships for the gradients and fluxes of mean and turbulent quantities in the CBL. The first of these, the convective turbulence scaling velocity

$$w_* \equiv (\beta z_i \overline{w'\theta'_0})^{1/3} , \quad (1)$$

determines much of CBL physics, including diffusion. Along with  $w'\theta'_0$ ,  $w_*$  forms the basis for the remaining parameters used to scale the heat, moisture, and momentum, as well as passive scalar fluxes. However,  $w_*$  neglects mechanical turbulence due to shear and is only strictly valid for free convection induced by surface heating. Yet,  $w_*$  is used often in mixed forced/free convection cases for lack of alternatives. Here we seek  $w_{*s}$ , an amendment to  $w_*$ , which includes mechanical turbulence contributions under mixed forced/free convection conditions. We assume mechanical generation comes from three sources: the barotropic, horizontally homogeneous surface layer (SL), barotropic boundary layer entrainment zone (EZ), and a baroclinity (BC) component. Beyond the  $z$  and  $z_i$  length scales, we propose that mixed forced/free convection also depends on the SL roughness length,  $z_0$ , SL Obukhov length,  $L \equiv -u_*^3 \theta / (gk w'\theta'_0)$  (where  $u_*$  is friction velocity and  $k \approx 0.4$  is the von Karmen constant), EZ thickness,  $\Delta h$ , and  $z_h$ , a baroclinic length scale which we define below. We model these mechanical sources, using relevant ratios of the length scales, thereby fulfilling similarity requirements.

## APPROACH

We outline the basic approach before discussing details. For non-baroclinic, horizontally homogeneous, steady state cases, the vertical profiles of the buoyancy and momentum fluxes must be linear, if the shapes of the vertical profiles of potential temperature and wind speed are to remain static. That is,

$$\overline{\beta w'\theta'_z} \approx \overline{\beta w'\theta'_0} (1 - (1 + a_1) z/z_i) , \quad (2a)$$

$$-\overline{u'_i w'_z} \approx -\overline{u'_i w'_0} (1 - (1 + a_2) z/z_i) , \quad (2b)$$

where  $a_1 = -\overline{w'\theta'_{zi}}/\overline{w'\theta'_0}$  and  $a_2 = \overline{u_i'w'_{zi}}/\overline{u_i'w'_0}$  (see fig. 1). The subscripts "zi" and "0" refer to conditions at the inversion base and surface.  $-\overline{u_i'w'_0} \equiv u_*^2$ ,  $w'\theta'_{zi} \sim We\Delta\theta$ , and  $-\overline{u_i'w'_{zi}} \equiv \Delta u_i We$ , where the CBL entrainment rate,  $We = z_{i,t}$  and  $\Delta\theta$  is the potential temperature jump across the EZ. The subscript "t" denotes the partial derivative with respect to time. Such notation will refer to spatial and temporal derivatives in general. The Betts (1974) correction,  $w'\theta'_{zi} \approx (\Delta\theta - \Gamma_u \Delta h)We$ , hardly affects eqn. 2a since Kamada (1988a) showed that the much smaller interfacial thickness,  $l_m$ , must replace the EZ depth,  $\Delta h$ . For lack of better information, we assume that above  $z_i$  the barotropic flux profiles are linear with height up to  $H \equiv z_i + \Delta h$ , the top of the boundary layer. I.e.,

$$\overline{\beta w'\theta'_z} \approx \overline{\beta w'\theta'_{zi}} \left(1 - \frac{z - z_i}{\Delta h}\right) \quad , \quad (3a)$$

$$-\overline{u_i'w'_z} \approx -\overline{u_i'w'_{zi}} \left(1 - \frac{z - z_i}{\Delta h}\right) \quad . \quad (3b)$$

Using eqns. (1), (2a), and (3a), the vertically integrated buoyancy flux or total CBL buoyancy production rate of turbulence kinetic energy (TKE) can be written to show that the purely free convective turbulent scaling velocity is proportional to the cube root of the boundary layer integrated TKE production rate,

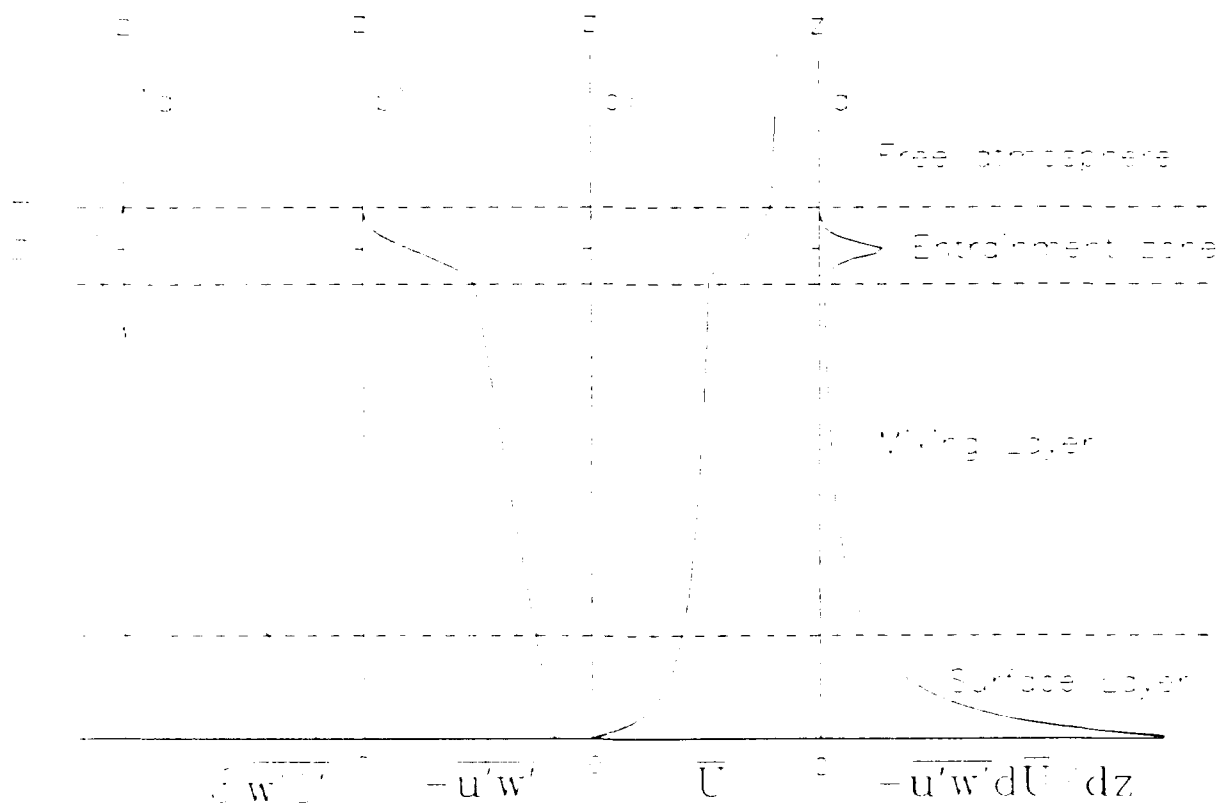
$$\int_{z_0}^H \overline{\beta w'\theta'_z} dz \approx \Omega w_*^3 \quad , \quad \text{where } \Omega \equiv \frac{1 - a_1(1 + \alpha)}{2} \quad , \quad (4)$$

and  $\alpha \equiv \Delta h/z_i$ .  $\Omega \approx 0.4$  for mature CBLs because under such conditions,  $a_1 \sim 0.2$  and  $\alpha \sim 0.1$ . Since this may not be true for other conditions, we discuss  $\Omega$  in detail below. However, here we propose that this sort of proportionality extends to shear; that mechanical contributions to a shear inclusive turbulent scaling velocity,  $w_{*s}$ , may be written in terms of the boundary layer integrated mechanical generation rate of TKE. I.e.,

$$w_{*s} \approx \left[ \frac{\int_{z_0}^H \overline{\beta w'\theta'_z} dz + \int_{z_0}^H -\overline{u_i'w'_{zi}} \partial_z dz}{\Omega} \right]^{1/3} \quad . \quad (5)$$

|                    |                                     |
|--------------------|-------------------------------------|
| Accession For      |                                     |
| NTIS GRA&I         | <input checked="" type="checkbox"/> |
| DTIC TAB           | <input type="checkbox"/>            |
| Unannounced        | <input type="checkbox"/>            |
| Justification      |                                     |
| By                 |                                     |
| Distribution/      |                                     |
| Availability Codes |                                     |
| Avail and/or       |                                     |
| Dist               | Special                             |
| A-1                |                                     |

fig.1 Schematic profiles of a) buoyancy, b) stress, c)  $\bar{U}$  velocity, and d) TKE shear generation rate for barotropic case



We also propose that the CBL shear generation rate can be represented by discrete barotropic surface layer (SL), entrainment zone (EZ), and baroclinic (BC) components.

$$\int_{z_0}^H \overline{-u'_i w' u_{i,z}} dz = \int_{z_0}^{-L} \overline{-u'_i w' u_{i,z}} dz + \int_{z_i}^{EZ} \overline{-u'_i w' u_{i,z}} dz + \int_0^H \overline{-u'_i w' u_{ib,z}} dz \quad (6)$$

Here  $u_{ib,z}$  is just the baroclinic portion of the vertical shear.

### BAROTROPIC SURFACE LAYER SHEAR

We first consider the barotropic SL and assume that most of the shear occurs below the height,  $-L$ . From SL similarity we have

$$u_{i,z} = (u_* / kz) \phi_m, \quad \text{with} \quad \phi_m = (1 - 15z/L)^{-1/3}, \quad (7a,b)$$

(Carl et al. 1973). Some workers prefer  $\phi_m$  formulas involving the  $-1/4$  power. However, the choice is among empirical expressions and integrals of  $-1/4$  power expressions result in awkward complex numbers. The accepted value in the free convection limit is  $-1/3$ , so we choose this for theoretical appeal as well as mathematical convenience.

Since  $\phi_m$  is only valid for  $z \gg z_0$ , we set the upper integration limit at  $-L$ , the lower limit at a somewhat arbitrary  $10 z_0$ , and treat the layer below  $10 z_0$  separately. Above  $10 z_0$ , from eqns. (2b) and (7a), we have

$$\begin{aligned} \int_{10z_0}^{-L} \overline{-u'_i w' u_{i,z}} dz &\approx \frac{u_*^3}{k} \int_{10z_0}^{-L} \frac{\phi_m \partial z}{z} + \frac{u_* (u_*^2 - \Delta u_i w_e)}{k z_i} \int_{10z_0}^{-L} \phi_m \partial z, \\ &= \frac{u_*^3}{k} \int_{10z_0}^{-L} \frac{\phi_m \partial z}{z} - (1-a_2) (L/z_{i2}) w_*^3 \int_{10z_0}^{-L} \phi_m \partial z. \end{aligned} \quad (8)$$

From the L definition,  $u_*^3/k = (-L/z_i) w_*^3$ . With the axes aligned with the mean wind, we have  $-u'_i w' u_{i,z} \approx -u' w' u_{z,z}$ , since  $(v_z, w_z) \ll u_z$  in the mixed layer. If shear supplies sufficient TKE,  $10 z_0 \ll -L$ . So in eqn. (8) term (2)  $\approx 0.54 (1 - a_2) (L/z_i)^2 w_*^3$ . Letting  $p = -15z/L$ , the first integral reduces to

$$\approx \sqrt{3} \operatorname{ATAN} \left[ \frac{\sqrt{3}(2(p+1)^{1/3} + 1)}{3} \right] - \frac{\operatorname{Ln}[(p+1)^{2/3} + (p+1)^{1/3} + 1]}{2} +$$

$$\operatorname{Ln}[(p+1)^{1/3} - 1] \Big|_{10z_0}^{-L}.$$

The upper limit evaluates to  $\approx 1.51$ . The first two terms in the lower limit  $\approx 1.26$ , if  $L \gg 150z_0$ . So in eqn. (8) the first integral  $\approx 0.25 - \operatorname{Ln} R$ , where  $R \equiv (1 - 150z_0/L)^{1/3} - 1 \ll 1$ . A 5cm  $z_0$  and 150m  $L$  would give  $150z_0 = -0.05L$ . Above  $10z_0$ , this leads to  $\approx 4.36u_*^3/k$  from term (1) in eqn. (8). Oddly,  $R$  varies inversely with roughness length. But from  $u_* = ku/(\ln(z/z_0 - \varphi_m))$  and the definition of  $L$ , we see that

$$nz_0/L = (\overline{ngw'\theta'}/k^2u_*^3\theta)z_0(\ln(z/z_0) - \varphi_m)^3. \quad (9)$$

So for given levels of  $u$ ,  $\overline{w'\theta'}$ ,  $\theta$ , and  $z$ , the ratio  $nz_0/L$  and thus term (1) may vary much less than  $z_0$  itself.

Below  $10z_0$  we neglect non-neutral contributions to  $u_*$  and assume a constant value,  $-u_*'w'u_{*z} \approx u_*^3/10kz_0$ . Then

$$\int_0^{10z_0} \overline{-u_*'w'u_{*z}} dz \approx u_*^3/k. \quad (10)$$

Together, eqns. (8) and (10) give an estimate of the vertically integrated SL shear production rate,

$$\begin{aligned} \int_0^{-L} \overline{-u_*'w'u_{*z}} dz &\approx (1.25 - \operatorname{Ln} R)u_*^3/k, \\ &\approx (-w_*^3L/z_1)(1.25 - \operatorname{Ln} R). \end{aligned} \quad (11a,b)$$

Note that we have dropped term (2) from eqn. (8), since 1) the coefficient is an order of magnitude smaller than for term (1), 2) term (2) is second order in  $z_1$ , 3) usually,  $-L < 0.4z_1$ , and 4)  $a_2$  is of order  $10^{-1} - 1$ ; so some further cancellation occurs.

From eqns. (2) and (3), if  $\Omega \approx 0.4$ , we have the following ratio of enhanced to unenhanced TKE contributions,



$$\left[ \frac{n - (1.25 - \ln R)L/z_i}{n} \right]^{1/3} w_* \approx (1 + (3.1 - 2.5 \ln R)(-L/z_i))^{1/3} w_* \quad (12)$$

For our previous example,  $w_{*s}$  exceeds  $w_*$  by a factor of  $\approx (1 + 33u_*^3/w_*^3)^{1/3}$ . In this case, since  $u_*/w_* \approx 0.54$ , SL shear generation alone may enhance  $w_{*s}$  by more than 80%.

To be useful,  $w_{*s}$  should maintain the same proportion to the mean turbulent velocity,  $\sigma_{ui}/3$ , in both the convective and neutral limits. We test this by noting for the barotropic case that the cube root of the vertically integrated production rate is usually  $\approx 2.4 u_*$  for SL shear. The EZ and BC terms are not relevant for the pure neutral case. From Grant (1986) and Mason and Thompson (1987), we find that  $\sigma_{ui}/3 \approx 1.3 u_*$  in the layer averaged neutral boundary layer. This implies that  $\sigma_{ui}/3 \approx 0.53 w_{*s}$ . In the pure convective limit, the CBL averaged value for  $\sigma_{ui}/3$  is  $\approx 0.55 w_*$  (Caughey and Palmer, 1979 and Deardorff et al. 1980). So, the ratio of  $w_{*s}$  to the layer averaged mean turbulent velocity is the same in the convective and neutral limits, as required.

#### ENTRAINMENT ZONE SHEAR

The EZ also generates mechanical turbulence. To quantify it we assume, moreover, that shear is constant across  $\Delta h$ . Thus,

$$\int_{z_i}^H -u_i'w'u_{i,z} dz \approx We \Delta u_i^2 / 2 \quad (13)$$

Equation (13) suggests the need for a simple shear inclusive model to estimate the entrainment rate,  $We$ . For this purpose we parameterize the TKE equation,

$$\begin{array}{cccccc} \text{storage} & \text{buoyancy} & \text{transport} & \text{shear} & \text{dissipation} & \\ e_{,i} = & \beta \overline{w'\theta'} + (\overline{u_i'e} + \overline{p'e/\rho})_{,z} & - \overline{u_i'u_j'u_{i,j}} & - \overline{v u_{i,j}u_{i,j}} & , & \end{array} \quad (14)$$

at the inversion base as,

$$\begin{array}{cccccc} \text{storage} & \text{buoyancy} & \text{transport} & \text{shear} & \text{dissipation} & \\ C_{st}w_*^2We/\Delta h \approx & -\beta \Delta \theta We & + C_c w_*^3/z_i & + C_m \Delta u_i^2 We/\Delta h & - C_d w_*^3/z_i & . \end{array} \quad (15)$$

The scaling arguments are that 1) again  $\overline{w'\theta'}_{zi} \approx -We\Delta\theta$  and 2) large mixed layer eddies drive the TKE transport rate into the EZ. Thus TKE transport should vary with the TKE ( $\propto w_*^2$ ) times the large eddy turnover time scale,  $z_i/w_*$ . 3) The shear production rate is modeled as the momentum flux,  $\Delta u_i We$ , times the local EZ shear,  $\Delta u_i/\Delta h$ . 4) To maintain a steady state inertial subrange, the dissipation plus buoyancy destruction rates must balance the large eddy TKE injection rate due to transport and shear production. So in the neutral limit, dissipation should scale like transport with the change rate of large eddy TKE,  $w_*^3/z_i$ . Since dissipation occurs at small scales, unaffected by buoyancy anisotropy, we retain the neutral form. For simplicity we neglect the small TKE storage rate term.  $C_m$  estimates vary somewhat, (Pollard et al. 1981; Kato and Phillips, 1967, Tennekes and Driedonks, 1981), but we see little reason for departures far from unity, so we assume  $C_m \sim 1.0$ . We discuss  $C_c$  and  $C_d$  below.

Tennekes and Driedonks (1981) presented a similar model, except that here we scale the shear production locally with  $\Delta h$  instead of  $z_i$ . Solving for the entrainment rate gives,

$$We = \frac{(C_c - C_d)}{\beta\Delta\theta - \Delta u_i^2/\Delta h} w_*^3/z_i \quad (16)$$

Therefore, from eqn. (13) we have

$$\int_{z_i}^H -u_i'w'u_{i,z} dz \approx \frac{(C_c - C_d)}{2(R_b - 1)} \frac{\Delta h}{z_i} w_*^3, \quad (17)$$

where the bulk Richardson number,

$$R_b \equiv \frac{\beta\Delta\theta\Delta h}{\Delta u_i^2} \quad (18)$$

The EZ is transitionally turbulent by nature. Thus, we expect  $R_b$  to fluctuate about some critical threshold turbulence value,  $R_{ch}$ .  $R_{ch}$  is much larger than the 1/4 value accepted for the stable SL because EZ TKE is mainly maintained by large eddy transport from below. So shear generation is only part of the TKE injection rate, and the dominant eddies are also much larger than in the transitionally turbulent SL. Thus, the EZ's large negative buoyancy will destroy much of the TKE during the eddy cascade before it can be dissipated at small scales. For these reasons

$R_{cb}$  may exceed unity in the EZ.

Indeed, our simulations to steady state using such an entrainment model suggest a typical value of  $R_{cb} \approx 1.3$ . From the results of Turner (1968), Deardorff et al. (1980), and many others, we suggest that  $C_c - C_d \approx 0.25$  in the convective limit. So in eqn. (17) the EZ shear contribution  $\approx 0.5 \alpha w_*^3$ , where  $\alpha \equiv \Delta h / z_i$ .

To assume that  $\alpha \approx 0.1 - 0.2$  may often suffice. Then,  $a_1 \approx 0.2$ ,  $\Omega \approx 0.4$ , and the barotropic EZ contribution to  $w_s$  is only a few percent. But  $\alpha$  can exceed unity during rapid entrainment into a near-neutral layer remaining from the previous day's CBL (Nelson et al. 1989). During such times, entrainment models suggest that  $a_1$  may exceed 40% (Kamada, 1988a). Then  $\Omega \approx 0.1$ , and EZ enhancement of  $w_s$  may exceed 0.4. With SL shear this may cause  $w_s$  to double. In this case a better diagnostic or prognostic estimate for  $\alpha$  is needed. For a diagnostic estimate, we begin by assuming  $R_b = R_{cb}$  and solve eqn. (17) for  $\Delta h$ . This leaves unknown  $\Delta u_i$  and  $\Delta \theta$ , the inversion temperature jump.  $\Delta \theta$  is hard to specify more precisely than  $\sim \pm 30\%$  because  $z_i$  and  $H$  are not that highly resolved. Yet, we know that  $\Delta \theta$  grows with the boundary layer growth rate,  $H_{i,t}$ , into the layer aloft (with lapse rate  $\Gamma_u$ ), and shrinks with the mixed layer warming rate,  $\Theta_{m,t}$ . So, using the simplest steady state assumption,

$$\Delta \theta_{i,t} = \Gamma_u H_{i,t} - \Theta_{m,t} = 0 \quad . \quad (19)$$

The mixed layer warming rate is given by the surface heating rate, plus the entrainment rate of warm air from aloft. i.e.,

$$\Theta_{m,t} = (\overline{w'\theta'}_0 - \overline{w'\theta'}_{z_i}) / z_i \quad . \quad (20)$$

So, if we approximate  $H_{i,t}$  as  $We(1 + \alpha)$ , we have

$$\Delta \theta = \overline{w'\theta'}_0 / We - \Gamma_u z_i (1 + \alpha) \quad . \quad (21)$$

Using eqns. (16) and (18) and combining constants, this results in two solutions for  $\alpha$ . We show only the physical root,

$$\alpha = \left[ \frac{2.3 \Delta u_i^2}{\Gamma_u \beta z_i^2} + \frac{1}{4} \right]^{1/2} - \frac{1}{2} \quad . \quad (22)$$

Though not explicit, eqn. (22) implicitly involves  $\Delta\theta$  because it presumes through eqns. (18 - 22) that  $\Delta\theta$  depends on  $\Gamma_u$ ,  $We$ ,  $w'\theta'_0$ , and  $z_i$ . This assumes that the capping inversion requires finite potential temperature and windspeed jumps, whenever there is horizontal wind.

In eqn. (22) we must still estimate the windspeed jumps across the EZ. Wyngaard (1988) proposed the following first order estimate. For steady flow over a horizontally homogeneous surface, the stress gradient is given by

$$-\overline{u'w'}_{,z} = f(v - v_g) \quad , \quad \text{and} \quad -\overline{v'w'}_{,z} = f(u_g - u) \quad , \quad (23a,b)$$

where  $f$  refers to Coriolis forcing, and the subscript,  $g$ , refers to geostrophic winds (Panofsky and Dutton, 1984). For barotropic flow with the axes aligned with the mean mixed-layer wind,

$$-\overline{u'w'}_{,z} = fv_g \approx (We(u_g - u) + u_*^2)/z_i \quad , \quad \text{and}$$

$$-\overline{v'w'}_{,z} = f(u_g - u) \approx We(v_g - v)/z_i \quad . \quad (24a,b)$$

The momentum balance becomes

$$We(u_g - u) + u_*^2 = -fz_iv_g \quad , \quad \text{and}$$

$$-We v_g = fz_i(u_g - u) \quad . \quad (25a,b)$$

For barotropic cases, if  $\Delta u \approx (u_g - u)$  and  $\Delta v \approx (v_g - v)$  across the EZ, then the solutions to eqns. (25a,b) are to first order,

$$\Delta u = We(u_*/fz_i)^2 \quad , \quad \text{and}$$

$$\Delta v = u_*^2/fz_i \quad . \quad (26a,b)$$

Here, almost all of the EZ shear results from turning the wind into the  $v$  direction. So for a mature mid-latitude BL with  $u_* = 0.4\text{ms}^{-1}$ ,  $f = 10^{-4}\text{s}^{-1}$ ,  $z_i = 10^3\text{m}$ ,  $\theta = 300^\circ\text{C}$ , and  $\Gamma_u = 3 \times 10^{-3}^\circ\text{C m}^{-1}$ , we have  $\alpha \approx 0.057$ . However, during growth into the near neutral remnant mentioned earlier, conditions may be more like  $\Gamma_u = 1 \times 10^{-3}^\circ\text{C m}^{-1}$ , and  $z_i = 500\text{m}$ . This would result in  $\alpha \approx 1.3$ . Values spanning such a range were reported in Nelson et al. (1989). They also observed and prognostically modeled the typical early morning to late afternoon hysteresis found in plots of  $\alpha$  versus  $We/w_*$ . They noted that such hysteresis is absent from previous

diagnostic expressions for the EZ depth which have the form  $\alpha \propto Ri_*^{-n}$ , where  $Ri_* \equiv \beta z_i \Delta \theta / w_*^2$  and  $1/4 \leq n \leq 1$ . No hysteresis occurs in such expressions, if we assume (see eqn. 2a) that  $a_1$ , the inversion to surface heat flux ratio, is constant because then  $We/w_* \propto 1/Ri_*$ . Unlike such expressions, eqn. (22) shows qualitatively appropriate hysteresis. In fact, by combining eqns. (16), (21), (22), and (26), we see that

$$\frac{\alpha}{We/w_*} \approx \frac{u_*^4 w_*}{w_*' \theta_*' \beta f^2 z_i^3} \quad (27)$$

Far from being constant,  $\alpha w_*/We$  will typically increase initially as surface heating and  $u_*$  grow in the young day-time boundary layer, but will diminish later as  $z_i$  becomes large.

If we require more accuracy, we can couple eqns. (16, 18, 19, 20, and 26) with

$$H_{*t} \approx We + (R_{ch}/\beta) \left[ \frac{2u_*^2 \Delta v We}{f z_i^2 \Delta \theta} - \frac{\Delta v^2 \Delta \theta_{*t}}{\Delta \theta^2} \right] \quad (28)$$

to provide a complete entrainment model. Here the second term in eqn. (28) gives the growth rate of  $\Delta h$  and thus a prognostic estimate for  $\alpha$ .

More physics may be added by revising the  $We$  expression to include the small TKE storage rate term,  $C_{st} w_*^2 / \Delta h$ , from eqn. (16). However, our present focus is amending  $w_*$ , rather than refining the particulars of bulk entrainment models; so we choose to rest with the above diagnostic and prognostic formulations for  $\alpha$ .

In either case, hysteresis in  $\alpha$  versus  $We/w_*$  is evident, and like Kamada (1988b, figs. 1 and 2), depends on changes in  $\Delta \theta$  and  $z_i$ . Both models include shear as well as heating due to entrainment, and avoids ad hoc surface temperature distributions, or fixed values of  $a_1$ . Also note that eqns. (16) and (21) lets us specify  $a_1$  which, together with eqn. (22), determines  $\alpha$  from eqn. (4).

## BAROCLINIC SHEAR

If the geostrophic wind varies linearly with height according to the thermal wind relations,  $u_{g,z} = -(g/fT_0)T_{*y}$ , and  $v_{g,z} = (g/fT_0)T_{*x}$ , then from eqn. (23) the baroclinic stress profile must be parabolic, rather than linear as in the barotropic case. Wyngaard (1985) gave the following simple estimate for baroclinic

shear based on the thermal wind,

$$u_{b,z} = b_1 u_{g,z} + b_2 v_{g,z} \quad , \quad v_{b,z} = b_2 u_{g,z} + b_1 v_{g,z} \quad , \quad (29a,b)$$

where  $b_1 = 1/(m^2 + 1)$ ,  $b_2 = m/(m^2 + 1)$ ,  $m = fz_i/w_*$ , and  $f$  is the Coriolis forcing. If so, the stress can be written as

$$\overline{-u_i'w'} = u_*^2 + \frac{b_2 z (gz \rho f T_{,y} + 2 f T p_{,y})}{2 \rho f T} \quad . \quad (30)$$

So the baroclinic shear contribution becomes

$$\int_0^H \overline{-u_i'w'} u_{i,z} dz \approx \frac{gz_i (b_2 T_{,x} - b_1 T_{,y}) (b_2 g \rho z_i^2 T_{,y} + 6 \rho T u_*^2)}{6 \rho f T^2} \quad , \quad (31)$$

where terms in  $b_1 - 1 = -m^2/(m^2 + 1)$  were considered neglectably small. The baroclinic integral is taken over the whole vertical range 0 to  $z_i$  rather than  $-L$  to  $z_i$  because the SL shear analysis only accounts for barotropic effects based on SL similarity. Scaling analysis of eqn. (31) shows that the squared and cubic terms in  $z_i$  are small. If so, to first order, the baroclinic shear contribution from eqn. (31) reduces to,

$$\int_0^H \overline{-u_i'w'} u_{i,z} dz \approx (z_i/z_b) w_*^3 \quad , \quad (32)$$

where we can define a baroclinic length scale as,

$$z_b \equiv \frac{f T w_*^3}{u_*^2 g (b_2 T_{,x} - b_1 T_{,y})} \approx \frac{w_*^3}{u_*^2 u_{b,z}} \quad . \quad (33)$$

Usually,  $z_b \gg z_i$ . But in coastal areas mesoscale temperature and pressure gradients are often large. For a coastal  $z_i$  of 300 m, a 3 °C/100 km horizontal cross-wind temperature gradient, We near zero,  $f = 10^{-4} \text{ sec}^{-1}$ ,  $w_* \approx 0.8 \text{ m s}^{-1}$ , and  $u_* \approx 0.4 \text{ m s}^{-1}$ , then  $z_b \approx 330 \text{ m}$ , and  $w_*$  may be enhanced baroclinically by ~ 50%.

## SUMMARY

In summary, we account for surface layer (SL), entrainment zone (EZ), and baroclinic (BC) shear. Each source is given in terms of a dimensionless ratio of the relevant length scales  $z_0$ ,  $L$ ,  $\Delta h$ ,  $z_i$ , and  $z_b$ . When summed and multiplied by  $w_*$ , this results in a turbulent velocity scale which includes both buoyancy and shear generation of TKE, namely,

$$w_{zs} \equiv \left[ \frac{\Omega - (1.25 - \ln R)L/z_i + 0.5\Delta h/z_i + z_i/z_b}{\Omega} \right]^{1/3} w_* \quad (34)$$

$$\text{Here } \Omega \equiv \frac{1 - a_1(1 + \alpha)}{2} ,$$

$$a_1 \approx We\Delta\theta/w'\theta'_0 , \quad \text{the } z_i/\text{surface heat flux ratio,}$$

$$We \approx \frac{(C_c - C_d)}{\beta\Delta\theta - \Delta u_i^2/\Delta h} w_*^3/z_i , \quad \text{the ML entrainment rate,}$$

$$\Delta\theta \approx \overline{w'\theta'_0}/We - \Gamma_u z_i(1 + \alpha) , \quad \text{the EZ } \theta \text{ jump,}$$

$$\alpha \equiv \Delta h/z_i \approx \left[ \frac{2.3(\Delta u^2 + \Delta v^2)}{\Gamma_u \beta z_i^2} + \frac{1}{4} \right]^{1/2} - \frac{1}{2} , \quad \text{the EZ/BL depth ratio,}$$

$$\Delta u \approx We(u_*/fz_i)^2 , \quad \Delta v \approx u_*^2/fz_i , \quad \text{the EZ velocity jump,}$$

$$R \equiv (1 - 150z_0/L)^{1/3} - 1 , \quad \text{an SL shear generation term,}$$

$$z_b \equiv \frac{w_*^3}{u_*^2 u_{b,z}} , \quad \text{the baroclinic length scale.}$$

## REFERENCES:

- Betts, A.K., 1974: Reply to comment on the paper, 'Non-precipitating cumulus convection and its parameterization'. Quart. J. Roy. Meteor. Soc., 100, 469-471.
- Businger, J.A., J.C. Wyngaard, Y. Izumi, and E.F. Bradley, 1971: Flux-profile relationship in the atmospheric surface layer. J. Atmos. Sci., 28, 181-189.
- Carl, M.D., T.C. Tarbell, and H.A. Panofsky, 1973: Profiles of wind and temperature from towers over homogeneous terrain. J. Atmos. Sci. 30, 788-794.
- Caughey, S.J., and S.G. Palmer, 1979: Some aspects of turbulence structure through the depth of the convective boundary layer.
- Deardorff, J.W., 1972: Numerical investigation of neutral and unstable planetary boundary layers. J. Atmos. Sci., 29, 91-115.
- Deardorff, J.W., G.E. Willis, and B.H. Stockton, 1980: Laboratory studies of the entrainment zone of a convectively mixed layer. J. Fluid Mech., 100, 41-64.
- Grant, A.L.M., 1986: Observations of boundary layer structure made during the 1981 KONTUR experiment. Quart. J. Roy. Meteor. Soc., 112, 825-841.
- Kamada, R.F., 1988: A fractal interfacial entrainment model for dry convective boundary layers. part I: Model description. J. Atmos. Sci., 45, 17, 2365-2374.
- , 1988: A fractal interfacial entrainment model for dry convective boundary layers. part II: Discussion of model behavior and comparison with other models. J. Atmos. Sci., 45, 17, 2375-2383.
- Kato, H. and O.M. Phillips, 1969: On the penetration of a turbulent layer into stratified fluid. J. Fluid. Mech. 37, 643-655.
- Mason, P.J. and D.J. Thompson, 1987: Large eddy simulations of the neutral-static-stability planetary boundary layer. Quart. J. Roy. Meteor. Soc., 113, 413-443.
- Mahrt, L., and D.H. Lenschow, 1976: Penetrative convection at the top of a growing boundary layer. J. Atmos. Sci., 33, 41-51.
- Nelson, E., R. Stull, and E. Eloranta, 1989: A prognostic relationship for entrainment zone thickness, J. Appl. Meteorol., 28, 885-903.



Panofsky, H.A., and J.A. Dutton, 1984: Atmospheric Turbulence. Wiley and Sons, New York, p. 133.

Pollard, R.T., P.B. Rhines, and R.O.R.Y. Thompson, 1973: The deepening of the wind-mixed layer', Geophys. Fluid Dyn. 3, 381-404.

Sorbjan, Z., 1989: Structure of the Atmospheric Boundary Layer. Prentice-Hall, Englewood Cliffs, New Jersey, p. 74.

Tennekes, H., and A.G.M. Driedonks, 1981: Basic entrainment equations for the atmospheric boundary layer. Bound.-Layer Meteorol., 20, 515-531.

Turner, J.S., 1968: The influence of molecular diffusivity on turbulent entrainment across a density interface. J. Fluid Mech. 23, 639-656.

Wyngaard, J.C., 1985: Structure of the planetary boundary layer and implications for its modeling. J. Clim. Appl. Meteorol., 24, 1131-1142.

———, 1988: Structure of the PBL. Lectures on Air Pollution Modeling, AMS, Boston, 1988, 25-29.

# DISTRIBUTION LIST

|     |   | No.copies |
|-----|---|-----------|
| 1.  | Defense Technical Information Center<br>Cameron Station<br>Alexandria, VA 22314                                       | 2         |
| 2.  | Dudley Knox Library<br>Naval Postgraduate School<br>Monterey, CA 93943-5000   | 2         |
| 3.  | Director of Research (08)<br>Naval Postgraduate School<br>Monterey, CA 93943-5000                                     | 1         |
| 4.  | Dept. of Physics (PH)<br>Naval Postgraduate School<br>Monterey, CA 93943-5000   | 2         |
| 5.  | Dr. Ray Kamada/061kd<br>Naval Postgraduate School<br>Monterey, CA 93943-5100  | 17        |
| 6.  | Capt. David Struck (SSD/CLGR)<br>Los Angeles AFB<br>Los Angeles, CA 90009-2960  | 1         |
| 7.  | Prof. Gordon Schacher<br>Dean of Faculty and Graduate Studies<br>Naval Postgraduate School<br>Monterey, CA 93943-5100 | 1         |
| 8.  | Capt. Mike Moss (AFESC/RDVS)<br>Tyndall AFB, FL 32403-6001  | 1         |
| 9.  | Lt. Col. Stephen Pryor (730 SPW)<br>Vandenberg AFB, CA 93437-5000   | 1         |
| 10. | Mr. Steve Sambol (730 SPW/DOW)<br>Vandenberg AFB, CA 93437-5000   | 1         |
| 11. | Mr. Randy Nyman<br>ACTA<br>Vandenberg AFB, CA 93437-5000  | 1         |
| 12. | Dr. Don Ermak<br>Lawrence Livermore National Lab<br>P.O. Box 808, L-262<br>Livermore, CA 94551                        | 1         |
| 13. | Mr. Bruce Kunkel (PL/GPAA)<br>Hanscom AFB, MA 01730-5000  | 1         |

14. Mr. Bill Boyd (45 WS/WER) 1  
Patrick AFB, FL 32925-5000
15. Mr. John Edwards 1  
SSD/DEV  
Los Angeles, AFB, CA 90009-2960
16. Mr. Larry Mendenhall 1  
Geodynamics Corp.  
21171 Western Ave  
Suite 100  
Torrence, CA 90501
17. Mr. Bart Lundblad (Aerospace/El Segundo) 1  
PO Box 92957  
Los Angeles, CA 90009
18. Mr. Jose Caraballo 1  
NASA  
Industrial Safety/RT-SOE  
KSC, Florida 39899
19. Mr. Jan Zysko 1  
NASA  
PM-PCO-4  
KSC, Florida 39899
20. Dr. Tetsuji Yamada 1  
Yamada Science and Art  
147 Monterey Bay Dr. S.  
Los Alamos, NM 87544
21. Dr. D. Pepper 1  
5301 N. Commerce #A  
Moorpark, CA 93021
22. Dr. Roger Gibbs 1  
Naval Surface Warfare Center  
Dahlgren, Virginia 22448-5000
23. Dr. Kenneth Moe 1  
USAF Space Division  
Box 92960 World Way  
Los Angeles, CA 90009-2960
24. Dr. Torben Mikkelsen 1  
Meteorologie und Vind Energie Dept.  
RISO National Laboratory  
DK-4000  
Roskilde, Denmark

25. Dr. Neals Otto Jensen 1  
Meteorologie und Vind Energie Dept.  
RISO National Laboratory  
DK-4000  
Roskilde, Denmark
26. Dr. John C. Wyngaard 1  
Dept. of Meteorology  
Pennsylvania State University  
University Park, PA 16802
27. Dr. Gary Geernert 1  
ONR, Office of the Chief of Naval Operations  
Arlington, VA 22217
28. Dr. Ted Tsui 1  
Naval Research Laboratory  
Monterey, CA 93943-5006
29. Dr. Steve Burke 1  
Naval Research Laboratory  
Monterey, CA 93943-5006
30. Dr. Paul Gudiksen 1  
Lawrence Livermore National Lab  
P.O. Box 808, L-262  
Livermore, CA 94551